Teacher Perspective "Squaring Off"

Class setup: Before class begins, the teacher needs to make and hang two posters. The first poster will be for recording the names of students assigned to each square size. It is helpful to have the names of the students posted for referencing during the lesson. The second poster will be used for recording the length, width, height, surface area and volume of each of the 11 boxes made during class. Depending on the students' prior knowledge, this lesson takes at least two class periods. Surface area has been focused on first, because students need more opportunity to make connections between the amount of paper used to make a box and comparing boxes of equal volume.

Class Poster #1

Assigned Square Side Size (inches)	**Student Name(s)**
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	
5.5	

Class Poster #2

Square Side Size (inch)	**Length (inch)**	**Width (inch)**	**Height (inch)**	**Surface Area (in^2)**	**Volume (in^3)**
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					
5.5					

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Box Models

Each student will make a box, but that is not the main goal of the exploration. The teacher needs to make a model of a rectangular box at several stages using 3 inches as the size of square removed. This will streamline the lesson's introduction and provide a classroom set of models for understanding the effect of zooming in later to find a better box. A class set of 11 different size boxes taped on the wall will serve as a visual for helping students see the effect that increasing and decreasing the square corner size has on the surface area and volume of the boxes made.

Box Model Stage A

- 1 -12in x 18in sheet colored construction paper
- 1 -12in x 18in sheet white construction paper

Box Model Stage B

- 1 - 12in x 18in sheet colored construction paper with a square cut from each corner
- 1 - 12in x 18in sheet white construction paper with the 4 cut out squares glued to each corner

Box Model Stage C

- 1 - 12in x 18in sheet colored construction paper with a square cut from each corner, crease the flaps into a box, but don't tape up the sides. Write the length, width, height dimensions on the flaps, calculate and write the area of the flap for the five flaps.
- 1 - 12in x 18in sheet white construction paper with the 4 cut out squares glued to each corner

Box Model Stage D

- 1 - 12in x 18in sheet colored construction paper with a square cut from each corner, crease the flaps into a box, tape up the sides and glue the colored box to the center of the white sheet of construction paper.
- Students record the length, width, height, and surface area on the class poster.
- Students tape their boxes in ascending or descending order on the wall.

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Materials:

Each student needs

- Squaring Off Guided Exploration
- 1 12in x 18in sheet colored construction paper
- 1 12in x 18in sheet white construction paper
- glue stick
- marker
- tape
- scissors
- ruler
- optional – calculator

Teacher needs

- Class poster “Square Side and Students”
- Class poster “Square Side, Length, Width, Height, Surface Area, Volume”
- Model of one box already made at different stages
- Overheads of each page of Squaring Off
- Optional - overhead calculator and viewscreen
- Overhead markers
- Markers for writing on class posters

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Guided Exploration:
Squaring Off Surface Area

Your boss at the local theater has given you the task of designing an open-topped rectangular box that will hold the most volume using a 12inch x 18inch sheet of paper. Your boss wants to use these boxes to advertise his promotions to win free tickets and snacks by purchasing something from the concession stand. If you come up with the right box dimensions for the graphic designer, your boss says he will give you a raise and you won't have to stay during cleanup after the late show on Saturday. You decide to get your math class involved to help design the right box. You have prepared the class with the following supplies, materials, questions, and tables to gather a whole class effort of finding the optimum box.

Materials: 1 white sheet and 1 colored sheet of 12inch x 18 inch construction paper, glue, tape, ruler, scissors, graph paper, calculator for each student.

Make a class set of open-topped boxes

Assign two or three students to each value from the chart below. These values each represent a side of a cut-out square from a corner of a 12in x 18in sheet of paper.

Square Side (inches)	Students
0.5 in	*Insert student names,*
1.0 in	*use a class poster*
1.5 in	*sheet for referencing*
2.0 in	*during lesson.*
2.5 in	
3.0 in	
3.5 in	
4.0 in	
4.5 in	
5.0 in	
5.5 in	

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Everyone Make Your Box!

1. Using your assigned value for the ***length of square side***, cut a square from each corner of a colored 12in x 18in sheet of construction paper. Draw a sketch of your colored pattern after the four squares are removed. Your drawn figure is called a ***net*** and represents the rectangular box in a ***two dimensional form***. *Students need to label width and length using an assigned square side value. A figure drawn below is the intended outcome for #1. During the next few steps in constructing the box, I have found it helpful to have several stages of a model box already done to streamline this piece of the lesson. It is vital that every student construct a box and that a set of 11 boxes are constructed for display on the classroom wall for further investigation into the lesson.*

2. Glue one of the cut out colored squares to each corner of a 12in x 18in white sheet of construction paper. *Refer to the teacher model.*

3. Fold up the flaps on the ***net*** piece to form an open-topped box in ***three dimensional form***. Tape the ***vertical*** edges together. Glue the box (open end up) to the center rectangular region of the white 12in x 18in sheet of construction paper. *Refer to the teacher model.*

4. Explain in words or mathematical symbols without measuring how to calculate the ***length***, ***width***, and ***height*** of your open-topped box. Label each edge in your model and each edge in the sketch above. Be sure to include ***appropriate units*** with each edge value. *Students need to see length expressed as a difference of 18 and 2 times a square side; also, width expressed as a difference between 12 and 2 times a square edge. If necessary, discuss order of operations. Students need to use their assigned square side values in explaining how they calculated their length, width, and height.*

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5. Explain in words, drawings or mathematical symbols how to calculate the ***surface area*** of your open-topped box. Be sure to include ***appropriate units*** for the surface area value. *Depending on the student group I am working with, I have found having the students label all the edges and faces of their constructed box helps in completing question #5. I have found two common approaches students take in calculating the surface area of their box. The most common is adding up the surface area of 5 faces. The least common is subtracting 4 squares from the original 12x18 sheet of paper. Discussing both methods, especially the least common method will eventually emerge from the lesson, but don't spill the beans until several students see this connection and then have them explain the connection.*

6. Compare your results with the other two students in your groups who also have the same length of square side. Record the edges and surface area values in the chart below. Someone in your group needs to record these values on the class data table. Be sure to include appropriate units for each value. *As students fill out the class chart, someone will notice a pattern of numbers emerging from the length, width, and height columns.*

Length =	18-2x (in)
Width =	12-2x (in)
Height =	x (in)
Surface Area =	(in^2)

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7. Using one of the boxes from your group, tape it on the wall with the other groups' boxes in order from smallest square removed to largest square removed. This is termed ***ascending order***. *At this point in the lesson, there should be an incredible display of 11 different size open-topped boxes lined up in ascending order. The remaining boxes the class has made may be arranged on the wall outside the classroom for viewing. I have displayed the boxes numerous ways, all with interesting visual effects.*

8. Look at the completed boxes taped to the wall. What do you notice about the relationship between the sides of the squares removed to the total surface area? Explain in words or mathematical symbols the relationship you noticed. *Possible student responses may include, as the size of the square increases the amount of paper left to make a box decreases; the surface area decreases; as the size of square increases, the height increases.*

9. Record another relationship that was noticed by the class.

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10. Use the class data table taped to the wall and complete the chart below.

Side of cut-out square (in)	Area of 1 cut-out square (in^2)	Area of 4 cut-out squares (in^2)	Surface Area (in^2)	Area of 4 Squares & Surface Area (in^2)
0.5 in	.25	1	215	216
1.0 in	1.00	4	212	216
1.5 in	2.25	9	207	216
2.0 in	4.00	16	200	216
2.5 in	6.25	25	191	216
3.0 in	9.00	36	180	216
3.5 in	12.25	49	167	216
4.0 in	16.00	64	152	216
4.5 in	20.25	81	135	216
5.0 in	25.00	100	116	216
5.5 in	30.25	121	95	216
X	X^2	$4x^2$	$216-4x^2$	216

11. In the table above, what pattern do you see for the total surface area of the rectangular box and the area of the 4 cut-out squares? Explain in words or mathematical symbols how to calculate the surface area of any box made by the class.

- *216 in^2 – 4 squares = Box surface area*
- *area of the rectangular box and 4 cut-out squares = 12inx18in sheet paper*
- *area of four smaller squares = one larger square*
- *12in x 18in sheet paper – one big square = area of rectangular box*

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12. Use the class data table taped to the wall and complete the table below.

Side of cut-out square (in)	Length (in)	Width (in)	Height (in)	Volume (in^3)
0.5 in	18-(2)(0.5)=17	12-(2)(0.5)=11	0.5	93.5
1.0 in	18-(2)(1.0)=16	12-(2)(1.0)=10	1.0	160.0
1.5 in	18-(2)(1.5)=15	12-(2)(1.5)=9	1.5	202.5
2.0 in	18-(2)(2.0)=14	12-(2)(2.0)=8	2.0	224.0
2.5 in	18-(2)(2.5)=13	12-(2)(2.5)=7	2.5	227.5
3.0 in	18-(2)(3.0)=12	12-(2)(3.0)=6	3.0	216.0
3.5 in	18-(2)(3.5)=11	12-(2)(3.5)=5	3.5	192.5
4.0 in	18-(2)(4.0)=10	12-(2)(4.0)=4	4.0	160.0
4.5 in	18-(2)(4.5)=9	12-(2)(4.5)=3	4.5	121.5
5.0 in	18-(2)(5.0)=8	12-(2)(5.0)=2	5.0	80.0
5.5 in	18-(2)(5.5)=7	12-(2)(5.5)=1	5.5	38.5
X	18-(2)(X) =	12-(2)(X) =	X	**(18-2x)(12-2x)(x)**

13. Look at the table above. Using words or mathematical symbols, describe the pattern in the table. Explain in words or mathematical symbols how to calculate the volume of any box made in class.

The length values decrease by 1 inch and the width values also decrease by 1 inch as the square side increases by ½ inch.

14. Explain in words, drawings or mathematical symbols how to calculate the ***volume*** of **your** open-topped box. Be sure to include ***appropriate units*** for the volume value.

Volume =(length x width x height) inch^3
V = (l)(w)(h) inch^3

15. Which box taped to the wall has the greatest volume? What are the dimensions of this box? *The box with greatest volume is the box with dimensions 13in x 7in x 2.5in. The volume in 227.5 in*^{3}

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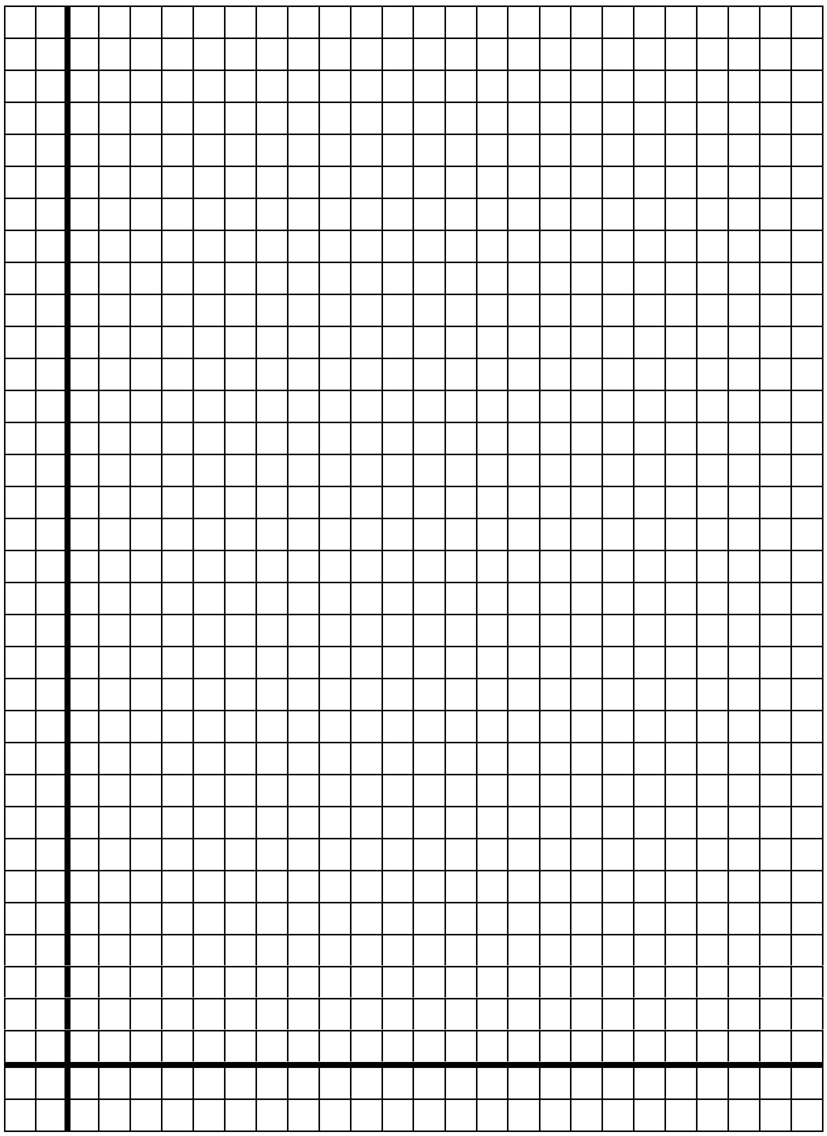
16. Is this the box with greatest volume? Explain how another box with greater volume than the one made in class, could be found. *To find a box with greater volume than 227.5 in^3, you need to look at square sizes left and right of 2.5 inches. The question is which side of 2.5 inches would lead to a box volume greater than 227.5 in^3.*

17. Graph the volumes of the 11 boxes built in class. Use the horizontal axis for cut-out square side and the vertical axis for volume. Label both axes.

Title:_____<u>Box Volume</u>__________



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18. Calculate the volume of four more boxes that will get closer to a box of greater volume than the box identified so far. Use the table below. Add these volumes to the graph. Which new square size results in a greater volume?

Side of cut-out square (in)	Length (in)	Width (in)	Height (in)	Volume (in^3)
2.0	18-(2)(2.0)=14.0	12-(2)(2.0)=8..0	2.0	224.00
2.1	18-(2)(2.1)=13.8	12-(2)(2.1)=7.8	2.1	226.04
2.2	18-(2)(2.2)=13.6	12-(2)(2.2)=7.6	2.2	227.39
2.3	18-(2)(2.3)=13.4	12-(2)(2.3)=7.4	2.3	228.07
2.4	18-(2)(2.4)=13.2	12-(2)(2.4)=7.2	2.4	228.10
2.5	18-(2)(2.5)=13.0	12-(2)(2.5)=7.0	2.5	227.50
X	18-(2)(X)=	12-(2)(X)=	X	

19. Using a 12in x 18in piece of paper, what is the equation for calculating the volume of any rectangular box? *Any version of multiplying length, width, and height is acceptable, such as*

$$V = (18-2x)(12-2x)(x)\ in^3$$
$$V = (12-2x)(18-2x)(x)\ in^3$$

20. Suppose an 8.5in x 11in paper was used instead of 12in x 18in. What would be the equation for calculating the volume of box using this new paper size?

$$V = (11-2x)(8.5-2x)(x)\ in^3$$

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